Test 1 Review

MTH1400 Online

Dr. Graham-Squire

•The test will cover sections 1.6, 1.7, 2.1-2.8, 3.1-3.6, and 4.1-4.4.

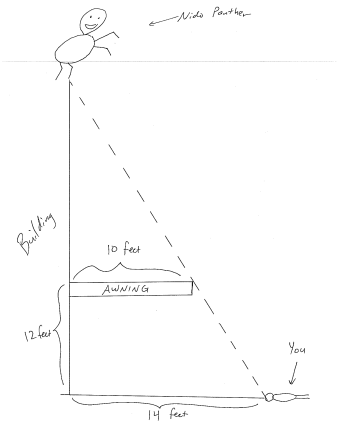
•To study, you can look over your notes, rework HW problems and quizzes on WebAssign, problems from the notes, as well as work out the practice problems given for each section. The Review Questions (in the textbook) at the end of Chapters 2, 3, and 4 are also good practice. You can also look at the following problems on my website:

- Fall 2013: Quizzes 1 through 4, Minitest 1, Test 2, and Test 3 numbers 1, 2, and 6 only.
- Summer 2009: Quiz 1 and Test 1 numbers 1 through 5 only.

•Calculators <u>are</u> allowed on this test, but for certain questions you may not be allowed to use a calculator. It is highly recommended that you bring a calculator because you cannot use cell phones or computers during the test.

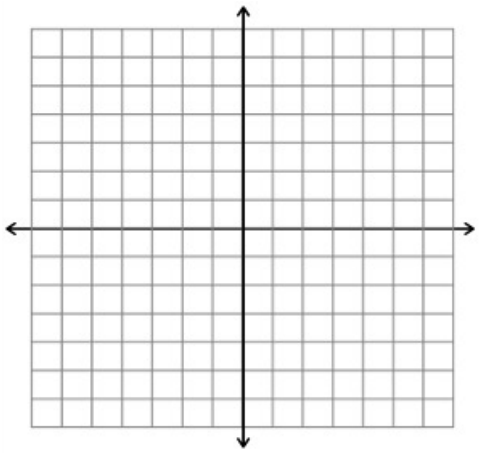
•Some practice problems to work on:

1. Jane invests \$1000 in a bank account at a certain interest rate, and \$3000 at an interest rate that is 3% higher. At the end of one year, she has earned \$270 from the two accounts. At what rate did she invest the \$1000?
2. HPU is constructing a new math building, and of course they want to make it as beautiful as possible. At the top of the building in the front entranceway they decide to build a giant panther statue with the head of Nido Qubein. You want to find out how tall the building is, so you do the following: From the front entrance, you walk out and lay on the ground at the point where your line of sight connects the awning with the base of the statue. If your head is 14 feet from the bottom of the building, the awning is 12 feet high, and the awning sticks out 10 feet horizontally from the building, find out how tall the building is.

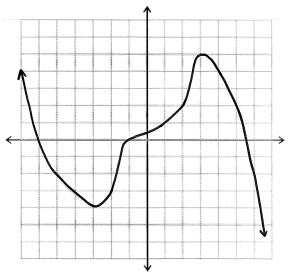


3. A hotel charges \$60 a night for the first three nights, and \$45 each additional night thereafter. Create a piecewise function $C(x)$ to represent the cost of staying x nights.

4. Sketch a graph of the piecewise function $g(x) = \begin{cases} -x & \text{if } x \leq 0 \\ 9 - x^2 & \text{if } 0 < x \leq 3 \\ x - 3 & \text{if } x > 3 \end{cases}$



5. Find all local maximums and minimums (and where they occur), as well as the intervals where the function is decreasing and where it is increasing.

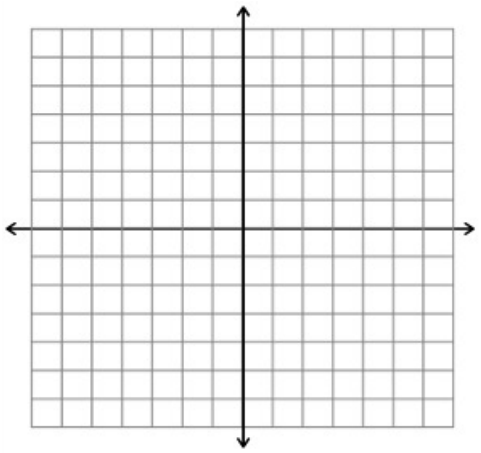


6. Find all local maximums and minimums of the function $g(x) = x^3 + 7x^2 - 30x$. Round to the nearest 0.1.

7. Find the average rate of change from $x = -5$ to $x = 4$ for the function in question 5.

8. Find the average rate of change from $x = 1$ to $x = 1 + h$ for the function in question 6.

9. Starting with the graph of $y = \sqrt{x}$, shift, flip and/or stretch the graph to find the graph of $y = -\sqrt{x+3} - 1$.



10. Let $f(x) = \dfrac{1}{\sqrt{x}}$, $g(x) = x^3$, and $h(x) = x^2 + 2x + 3$. Find the composition of functions $g(h(f(x)))$. Simplify your answer if possible.

11. Graph each function to confirm that it is one-to-one. If it is not, restrict the domain so that it is one-to-one. Then find the inverse for each function.

    (a) $f(x) = \dfrac{2-3x}{4+x}$

    (b) $f(x) = \dfrac{1}{2}(x-7)^2 + 3$

12. The owner of a luxury motor yacht that sails among the Greek islands charges \$600/person if exactly 20 people sign up for the cruise (which gives a total revenue of $600 \cdot 20 = 12,000$ dollars) . However, if more than 20 people sign up (up to the maximum capacity of 90) for the cruise, then each fare is reduced by \$4 for each additional passenger. Thus if there are 21 people, the fare is \$596 per person for everyone, for a total revenue of $596 \cdot 21 = 12516$ dollars. If there are 22 people then the fare is \$592 per person, etc. Assuming at least 20 people sign up for the cruise, let x be the number of passengers over 20 who sign up for the cruise. Answer the following questions:

    (a) Construct a quadratic function $r(x)$ to represent the total revenue in terms of x.

    (b) Find the number of passengers that would give the *maximum* amount of revenue for the owner of the yacht.

    (c) What is the maximum revenue possible?

    (d) Explain how you could find (b) and (c) using a graph of $r(x)$, and how you would find them without using a graph.

13. Find the quotient and remainder for the division $\dfrac{x^5 - 2x^4 + x^3 - 3x + 1}{x^2 + 4x - 1}$

14. An open box with a volume of 1500 cm^3 is to be constructed by taking a piece of cardboard 20 cm by 40 cm, cutting squares of length x from each corner, and folding up the sides. Show that this can be done in two different ways and find the exact value of x in each case. Note that "exact" means that I do not just want a decimal approximation!

    (a) Draw a diagram of the situation. Start with a 20×40 rectangle, then show how the corners are cut out, then show how it is folded up into a box.

(b) Write an equation (in terms of x) that represents the volume of box, and then move stuff to one side so that you have a polynomial equal to zero.

(c) Use graphing and/or factoring techniques to find all the zeroes of the polynomial from part (b).

15. Find all zeroes for the following polynomials, both real and complex:

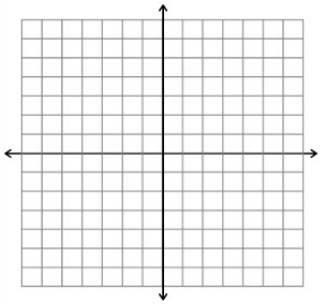
(a) $x^5 + 5x^3 - 36x$

(b) $x^5 - 2x^4 + x^3 - 8x^2 + 16x - 8$ (Hint: try factoring by grouping- You will have to group in a different manner than previous problems, though, possibly rearranging some of the terms).

16. (a) Find all x and y intercepts and all vertical and horizontal asymptotes for the rational function

$$f(x) = \frac{-2(x-3)(x+3)}{x(x+5)(x-2)}$$

(b)Sketch the graph of $f(x)$.



17. Bob invests \$10,000 in a bank account at 4% interest, compounded continuously. How much money will he have in the account after 5 years? Round to the nearest dollar.

18. Radioactive iodine is used as a tracer to diagnose certain thyroid gland disorders. It decays in such a way that the mass (in grams) remaining after t days is given by the exponential decay function, with $P = 6$ and $r = -0.087$. Answer the first two questions *without* using a calculator. You will need a calculator to answer the third question.

(a) How much of the iodine is present initially?
(b) How much iodine will be left in the body over the long run (that is, as t goes to infinity)?
(c) How long does it take for the half of the iodine to leave the body?

19. Use the definition of logarithm to solve the equations. You should be able to do these without a calculator.

(a) $\log_4 2 = x$
(b) $\log_4 x = 2$
(c) Evaluate $\log_5 100 - \log_5 10 + \log_5 5 - \log_5 2$

20. Use laws of logarithms to completely expand the expression $\ln\left(\dfrac{e^x}{x(x^2+1)(x^4+1)}\right)$.